A NEW RESULT ON AVERAGING THEORY FOR A CLASS OF DISCONTINUOUS PLANAR DIFFERENTIAL SYSTEMS WITH APPLICATIONS

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Abstract. We develop the averaging theory at any order for computing the periodic solutions of discontinuous piecewise differential system of the form

$$r' = \begin{cases} F^+(\theta, r, \varepsilon) & \text{if} \quad 0 \le \theta \le \alpha, \\ F^-(\theta, r, \varepsilon) & \text{if} \quad \alpha \le \theta \le 2\pi, \end{cases}$$

where $F^{\pm}(\theta, r, \varepsilon) = \sum_{i=1}^{k} \varepsilon^i F_i^{\pm}(\theta, r) + \varepsilon^{k+1} R^{\pm}(\theta, r, \varepsilon)$ with $\theta \in \mathbb{S}^1$ and $r \in D$, where D is an open interval of $\mathbb{R}^+$, and ϵ is a small real parameter.
Applying this theory, we provide lower bounds for the maximum number of limit cycles that bifurcate from the origin of quartic polynomial differential systems of the form $\dot{x} = -y + xp(x, y)$, $\quad \dot{y} = x + yp(x, y)$, with $p(x, y)$ a polynomial of degree 3 without constant term, when they are perturbed, either inside the class of all continuous quartic polynomial differential systems, or inside the class of all discontinuous piecewise quartic polynomial differential systems with two zones separated by the straight line $y = 0$.

1. Introduction and Statement of the Main Results

One of the main open problems in the qualitative theory of polynomial differential systems in $\mathbb{R}^2$ is the determination of their limit cycles, see for instance [10]. Bifurcations of limit cycles have been exhaustively studied in the last century and is closely related to the Hilbert's 16^{th} problem. However, in spite of all efforts, up to now there is no general method to solve this question.

Bifurcation of limit cycles in continuous planar differential systems are still largely studied. Nonetheless, due to the considerable number of discontinuous phenomena in the real world, see for example [5, 20] and the references therein, a significant interest in the investigation of limit cycles of discontinuous piecewise differential systems has arisen. For instance in [17], applying the theory of regularization, the averaging theory is extended up to order 1 for studying the

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