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ON THE CONNECTIVITY OF THE ESCAPING SET FOR COMPLEX EXPONENTIAL MISIUREWICZ PARAMETERS

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ABSTRACT. Let $E_\lambda(z) = \lambda\exp(z), \lambda \in \mathbb{C}$, be the complex exponential family. For all functions in the family there is a unique asymptotic value at 0 (and no critical values). For a fixed λ, the set of points in $\mathbb{C}$ with orbit tending to infinity is called the escaping set. We prove that the escaping set of E_λ with λ Misiurewicz (that is, a parameter for which the orbit of the singular value is strictly preperiodic) is a connected set.

1. Introduction

For an entire function $f : \mathbb{C} \to \mathbb{C}$ there are two types of points into which some branches of f^{-1} cannot be continued analytically; these are critical values and asymptotic values. A *critical value* is the image of a critical point (a zero of the derivative of f), and $z_0 \in \mathbb{C}$ is an *asymptotic value* if there is a curve $\alpha(t)$ satisfying $|\alpha(t)| \to \infty$ and $f(\alpha(t)) \to z_0$, as $t \to \infty$. The closure of the union of critical and asymptotic values is called the set of *singular values*. Singular values are known to play an important role in determining the global dynamics associated with the iterates of the map; see [4] (Chapter III) or [2] (Theorem 7).

Let $(E_\lambda)_{\lambda\in\mathbb{C}}$ be the complex exponential family, i.e., $E_\lambda(z) = \lambda\exp(z)$, $\lambda \in \mathbb{C}$. Each complex exponential map is a transcendental entire map with a unique asymptotic value at $z = 0$ and no critical values. Therefore, the structure and topology of the Fatou and Julia sets in the dynamical plane strongly depend on the asymptotic behavior of the iterates of the unique singular value at $z = 0$. For this reason E_λ is considered the transcendental entire version of the well–known quadratic polynomial family, $Q_c(z) = z^2 + c$, $c \in \mathbb{C}$, which has, for each c, a unique critical value at $z = c$ (and, of course, no asymptotic values) which determines the structure and topology of the Fatou and Julia sets in the dynamical plane.

However, in contrast to the polynomial case, where all points that tend to infinity under iteration belong to the basin of attraction of $z = \infty$ and therefore belong to the Fatou set, the existence of an essential singularity at infinity and a unique finite singular value implies that all points that tend to infinity under iteration, known

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