GLOBAL STRUCTURAL STABILITY OF PLANAR HAMILTONIAN VECTOR FIELDS*

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Abstract. We characterize all structurally stable C^r planar Hamiltonian vector fields with respect to perturbations in the set of all C^r planar vector fields. Also, we extend this characterization to C^r planar integrable vector fields, and we show that the usual requirement that the homeomorphism lies near the identity in the definition of structural stability is redundant for the C^r planar Hamiltonian vector fields.

1. Introduction and statement of the results. The global structural stability of C^r Hamiltonian vector fields with n degree of freedom is a hard open problem. In this paper we will characterize the C^r planar Hamiltonian vector fields (i.e., with 1 degree of freedom) which are structurally stable with respect to perturbations in the set of all C^r planar vector fields. Furthermore, we extend this characterization to C^r planar integrable vector fields.

In [JL] we classify the structural stability of a planar Hamiltonian polynomial vector field with respect to perturbations, first in the set of all C^r planar vector fields, second in the set of all planar polynomial vector fields, and finally in the set of all planar Hamiltonian polynomial vector fields. A summary of these results is given in the Appendix.

In dimension 2 the study of the structural stability of vector fields goes back to Andronov and Pontrjagin [AP] who in 1937 studied the structural stability of analytic vector fields on the closed 2-dimensional disk. Peixoto [P] in 1962 characterized the C^1 vector fields defined on the orientable differentiable compact connected 2-manifolds without boundary which are structurally stable. In 1982 Kotus, Krych and Nitecki [KKN] gave sufficient conditions in order to a C^r vector field on an orientable differentiable open connected 2-manifold without boundary S to be structurally stable (furthermore they proved that these conditions are necessary if $S = \mathbb{R}^2$). Other papers related with the structural stability of some classes of planar vector fields on are due to Sotomayor [So2] and Shafer [S1],[S2].

In order to state our results we will need some definitions and notation. Since a C^r planar vector field X can be regarded as a C^r map from $\mathbb{R}^2$ to $\mathbb{R}^2$, we can endow the space of all C^r vector fields with the *strong C^r-topology or Whitney C^r-topology*. This topology permits (in contrast with the *compact-open topology*) the control of the behaviour of perturbations of $X(p)$ when p goes to infinity. We denote by $\mathcal{X}^r, r \geq 1$, all the C^r vector fields on $\mathbb{R}^2$

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