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Hamiltonian stability in the plane

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Abstract. We characterize the Hamiltonian flows in the plane which are structurally stable (in a global sense) among Hamiltonian flows. This notion is closely related to, but distinct from, the topological stability of the generating function as a map from the plane to the line.

1. *Introduction and statement of results*

In this paper we give a dynamic characterization of the Hamiltonian flows whose phase space is the plane $\mathbb{R}^2$ which possess a certain global kind of structural stability.

The essentially unique topology for the space of continuous functions on a compact space leads to a natural notion of structural stability for dynamical systems on a closed manifold. However, the continuous functions on a non-compact space have a number of natural topologies. As a result, several distinct versions of structural stability can be formulated for dynamical systems on an open manifold, such as the plane. The comparative discussion of these notions in [**KKN**] gives a rationale for the version we shall adopt here.

For r a non-negative integer, $\mathcal{F}^r$ denotes the set of $\mathcal{C}^r$ functions $f : \mathbb{R}^2 \to \mathbb{R}$. Given $f \in \mathcal{F}^r$ and $x \in \mathbb{R}^2$, let $\|f(x)\|_{\mathcal{C}^r}$ be the maximum among the absolute values of $f(x)$ and its partial derivatives up to and including order r, all evaluated at x. A basis for the neighborhoods of $f \in \mathcal{F}^r$ in the *strong $\mathcal{C}^r$ topology* of Whitney is given by the sets

$$\mathcal{N}_\varepsilon(f) = \{g \in \mathcal{F}^r \mid \|g(x) - f(x)\|_{\mathcal{C}^r} < \varepsilon(x)\ \forall x \in \mathbb{R}^2\}$$

where $\varepsilon : \mathbb{R}^2 \to \mathbb{R}^+ = (0, \infty)$ ranges over positive functions on $\mathbb{R}^2$. The space $\mathcal{X}^r$ of $\mathcal{C}^r$ vectorfields is topologized by applying these estimates componentwise.

A *dynamical equivalence* between two flows Φ and Ψ on $\mathbb{R}^2$ is a homeomorphism $h : \mathbb{R}^2 \to \mathbb{R}^2$ taking directed Φ-trajectories to directed Ψ-trajectories. Given $K \subset U \subset \mathbb{R}^2$

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