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Periodic orbits and nonintegrability of generalized classical Yang–Mills Hamiltonian systems

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The averaging theory of first order is applied to study a generalized Yang–Mills system with two parameters. Two main results are proved. First, we provide sufficient conditions on the two parameters of the generalized system to guarantee the existence of continuous families of isolated periodic orbits parameterized by the energy, and these families are given up to first order in a small parameter. Second, we prove that for the nonintegrable classical Yang–Mills Hamiltonian systems, in the sense of Liouville–Arnold, which have the isolated periodic orbits found with averaging theory, cannot exist in any second first integral of class $\mathcal{C}^1$. This is important because most of the results about integrability deals with analytic or meromorphic integrals of motion. © *2011 American Institute of Physics*. [doi:10.1063/1.3559145]

I. INTRODUCTION

We study a *generalized classical Yang–Mills (YM) Hamiltonian*,[25] which consists of a harmonic oscillator plus a homogeneous potential of fourth degree:

$$H = \frac{1}{2}(p_x^2 + p_y^2 + x^2 + y^2) + \frac{a}{4}x^4 + \frac{b}{2}x^2y^2, \tag{1}$$

with two real parameters a and b. When $a = 0$ we obtain the *Contopoulos Hamiltonian*, studied by him and co-workers during many years, see for instance Refs. 11, 12, and 13. The Contopoulos Hamiltonian describes the perturbed central part of an elliptical or barred galaxy without escapes. When in the Contopoulos Hamiltonian, the quadratic part $(x^2 + y^2)/2$ is not present we have the *mechanical Yang–Mills Hamiltonian* $H_{YM} = (p_x^2 + p_y^2)/2 + b\,x^2y^2/4$; the term x^2y^2 characterizes the Yang–Mills potential, which arises in connection with the classical Yang–Mills field with gauge group SU(2) for a homogeneous two-component field.[18] Quartic homogeneous potentials (without quadratic terms) have been studied by several authors, see for instance Refs. 2, 5, and 16 and it is well known that the Hamiltonian H_{YM} with $b \neq 0$ is nonintegrable and strongly chaotic. Generalizations of the mechanical Yang–Mills Hamiltonian, with three up to five quartic terms, have been considered in Refs. 9, 15, 23, and 25. Maciejewski *et. al.*[25] studied generalized Yang–Mills Hamiltonian systems, which have a quadratic potential plus a homogeneous of fourth degree potential with five parameters, and they proved the existence of connected branches of nonstationary periodic trajectories emanating from the origin. Caranicolas *and Varvoglis*[9] studied a Hamiltonian with a quartic potential of three parameters plus a quadratic harmonic potential with frequencies ω_1 and ω_2 being two extra parameters of the form

$$H = \frac{1}{2}(p_x^2 + p_y^2 + \omega_1^2x^2 + \omega_2^2y^2) + \varepsilon(a\,x^4 + 2b\,x^2y^2 + c\,y^4), \tag{2}$$

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