JOURNAL OF MATHEMATICAL PHYSICS **53**, 042901 (2012)

The cored and logarithm galactic potentials: Periodic orbits and integrability

Lidia Jiménez-Lara[1,a)] and Jaume Llibre[2,b)]
[1]*Departamento de Física, Universidad Autónoma Metropolitana-Iztapalapa, P.O. Box 55-534, México, D.F., 09340 México*
[2]*Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain*

(Received 14 December 2011; accepted 4 March 2012; published online 6 April 2012)

We apply the averaging theory of first order to study analytically families of periodic orbits for the cored and logarithmic Hamiltonians $H_C = \frac{1}{2}\left(p_x^2 + p_y^2\right) + \sqrt{1 + x^2 + \frac{y^2}{q^2}} - 1$, and $H_L = \frac{1}{2}\left(p_x^2 + p_y^2\right) + \frac{1}{2}\log\left(1 + x^2 + \frac{y^2}{q^2}\right)$, which are relevant in the study of the galactic dynamic. We first show, after introducing a scale transformation in the coordinates and momenta with a parameter ε, that both systems give essentially the same set of equations of motion up to first order in ε. Then the conditions for finding families of periodic orbits, using the averaging theory up to first order in ε, apply equally for both systems in every energy level $H = h > 0$ with H either H_C or H_L. We prove the existence of two periodic orbits if q is irrational, for ε small enough, and we give an analytic approximation for the initial conditions of these periodic orbits. Finally, the previous periodic orbits provide information about the non-integrability of the cored and the logarithmic Hamiltonian systems. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.3697838]

I. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

We study the following two potentials:

$$V_C = \sqrt{R^2 + x^2 + y^2/q^2} - R,$$
$$V_L = \frac{1}{2}\log(R^2 + x^2 + y^2/q^2), \tag{1}$$

called the *cored and logarithmic potentials*, respectively, which have an absolute minimum and reflection symmetry with respect to both axes. These potentials are of interest in problems of galactic dynamics as models for elliptical galaxies.

The logarithmic potential is a model of a core embedded in a dark matter halo, with R the core radius.[11] Papaphilippou and Laskar[10] use the Laskar's frequency map analysis applied to study the numerically integrated orbits, giving a global vision of the dynamics in the phase space. Contopoulos and Seimenis[6] have applied the Prendergast method to approximate solutions of the logarithmic potential in the form of rational functions. An analogous method to the Pendergrast based in series expansions computed by inverting the normalizing canonical transformation was used by Pucacco *et al.*[12] to find periodic orbits. The structure of the phase space related to the logarithmic potential has been approximated with resonant detuned normal forms constructed with the method based on the Lie transform by Pucacco *et al.*[11,12] and Belmonte *et al.*[4] Detuned resonant

a)Electronic mail: lidia@xanum.uam.mx.
b)Elctronic mail: jllibre@mat.uab.cat.

