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# Limit Cycles of Planar Discontinuous Piecewise Linear Hamiltonian Systems Without Equilibria Separated by Nonregular Curves

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The problem of determining the existence, maximum number and positions of the limit cycles of the planar discontinuous piecewise linear differential systems is an important problem in the qualitative theory of differential systems. In this paper, we study two families of piecewise linear Hamiltonian systems without equilibria in $\mathbb{R}^2$ separated by a nonregular curve. We provide the maximum number of crossing limit cycles that each family can have and show when this maximum is reached. In this way we are solving for each family the extended 16th Hilbert problem.

*Keywords*: Crossing limit cycle; discontinuous piecewise linear Hamiltonian system; nonregular curve.

## 1. Introduction and Statement of the Main Results

A *discontinuous piecewise differential system* on $\mathbb{R}^2$ is a pair of $C^r$ (with $r \geq 1$) differential systems in $\mathbb{R}^2$ separated by a smooth curve $\Sigma$. The *line of discontinuity* $\Sigma$ of the discontinuous piecewise differential system is given by $\Sigma = h^{-1}(0)$, where $h : \mathbb{R}^2 \to \mathbb{R}$ is a $C^1$ function having 0 as a regular value. Observe that $\Sigma$ is the boundary between the regions $\Sigma^+ = \{(x,y) \in \mathbb{R}^2 \,|\, h(x,y) > 0\}$ and $\Sigma^- = \{(x,y) \in \mathbb{R}^2 \,|\, h(x,y) < 0\}$. Hence

$$Z(x,y) = \begin{cases} X(x,y), & \text{if } h(x,y) \geq 0, \\ Y(x,y), & \text{if } h(x,y) \leq 0, \end{cases} \tag{1}$$

is the vector field corresponding to a piecewise differential system with line of discontinuity $\Sigma$.

When the vector fields $X$ and $Y$ coincide on the line $\Sigma$ we have a *continuous piecewise differential system* on $\mathbb{R}^2$, that in general will not be smooth on $\Sigma$.