PHYSICAL REVIEW E **83**, 016110 (2011)

Uncorrelatedness in growing networks with preferential survival of nodes

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(Received 5 September 2010; revised manuscript received 24 November 2010; published 27 January 2011)

The emergence of uncorrelated growing networks is proved when nodes are removed either uniformly or under the preferential survival rule recently observed in the World Wide Web evolution. To this aim, the rate equation for the joint probability of degrees is derived, and stationary symmetrical solutions are obtained, by passing to the continuum limit. When a uniformly random removal of extant nodes and linear preferential attachment of new nodes are at work, we prove that the only stationary solution corresponds to uncorrelated networks for any removal rate $r \in (0,1)$. In the more general case of preferential survival of nodes, uncorrelated solutions are also obtained. These results generalize the uncorrelatedness displayed by the (undirected) Barabási-Albert network model to models with uniformly random and selective (against low degrees) removal of nodes.

DOI: 10.1103/PhysRevE.83.016110 PACS number(s): 89.75.Hc, 89.75.Fb, 87.23.−n

I. INTRODUCTION

Since the publication of the influential papers [1,2] on complex networks, this topic have attracted a good deal of attention from scientists in different fields. From the beginning, a remarkable effort has been devoted to predict the implications of different mechanisms for the addition and removal of nodes for the structure and functionality of the resulting network. Selective node deletion, affecting mainly nodes with a high connectivity, has been associated with intentional attacks, whereas uniformly random deletion is associated with random failures of nodes [3,4]. Moreover, a recent empirical study on time evolution of large data sets of the World Wide Web (WWW) concludes that, considering the WWW as an undirected network, the deletion probability of a given page (node) decreases with the number of links to other pages (node degree) according to a power law [5]. This suggests to the authors that a new mechanism, called *preferential survival*, is at work in the evolution of the WWW. The same mechanism also has been introduced recently in [6] for a food-web evolutionary model, and a somewhat similar mechanism was proposed in [7] for the deactivation probability of nodes in citation networks.

The consequences of a uniformly random deletion have been widely analyzed for different models, and some of its implications are well understood [8–13]. On the contrary, network models involving selective removal are far more complicated to analyze [14], and some caution must be taken before trusting some results extracted from models in which correlations have not been considered (see, for instance, [5]). The main difficulty in solving these models lies in the fact that the consequences of deletion on the degree evolution of nodes can not be evaluated without *a priori* knowledge of some topological features of the network, such as, in particular, the degree correlations. An especially simple case that allows us to overcome this difficulty is given by the assumption that correlations are absent (uncorrelated networks). Of course, one has to prove that such an assumption remains true as the network evolves, which, in general, will not be the case. Indeed, a revealing recent work has shown that negative degree-degree correlations in highly heterogeneous real networks are to be expected in the absence of specific constraints [15]. In particular, for some empirical scale-free networks, the negative value of the Pearson's coefficient for the degree correlation is close to the value of the configuration of maximum total entropy of the expected values of the elements of the adjacency matrix for a given degree sequence $\{k_i\}_{i=1}^N$, under the assumption of a scale-free degree distribution. In this sense, other previous works [16,17] showed that having at most one link per pair of nodes induces some negative correlation, which, in general, is weaker than those observed in real networks [15]. Hence, for some scale-free networks, negative correlations can be explained without invoking anticorrelating mechanisms. In the framework of evolving networks under the selective removal of nodes, the interplay between the degree evolution and degree correlations comes into play and offers a challenging problem when one aims to explain the origin of topological features such as degree correlations. Certainly, stationary networks arising from mechanisms of addition and deletion of nodes will not fulfill any principle of maximum entropy. Hence, a first step is to see which mechanisms acting during the network growth are (or are not) able to render correlated networks.

II. THE DISCRETE RATE EQUATION FOR e_{ij}

Let us consider a general growing network that evolves, at each time $t > 0$, under the addition and deletion of nodes. Precisely, a new node is connected to the network by means of m new links. Each new link attaches to an extant node with a probability that depends only on its degree k. Following [14], this probability will be denoted by $\Pi_k^a(t)$. Moreover, with probability $0 \leqslant r < 1$, a node is removed from the network (and all the edges incident to the removed node are also removed). A particular node will be removed with a probability $\Pi_k^d(t)$, which depends only on its degree k. If $n_k(t)$ is the number of nodes with degree k at time t in the network, then $\Pi_k^a(t)\, n_k(t)$ and $\Pi_k^d(t)\, n_k(t)$ are, respectively, the probability that the newly added node is connected to some node of degree k and the probability of deleting some node of degree k (provided a removal occurs, with probability r).

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