Tuning the overlap and the cross-layer correlations in two-layer networks: application to an SIR model with awareness dissemination

David Juher*
Department of IMAE, Universitat de Girona, Catalonia, Spain

Joan Saldaña†
Department of IMAE, Universitat de Girona, Catalonia, Spain and
Department of Electrical and Computer Engineering, Kansas State University, Kansas, USA
(Dated: February 13, 2018)

We study the properties of the potential overlap between two networks A, B sharing the same set of N nodes (a two-layer network) whose respective degree distributions $p_A(k), p_B(k)$ are given. Defining the overlap coefficient α as the Jaccard index, we prove that α is very close to 0 when A and B are random and independently generated. We derive an upper bound α_M for the maximum overlap coefficient permitted in terms of $p_A(k)$, $p_B(k)$ and N. Then we present an algorithm based on cross-rewiring of links to obtain a two-layer network with any prescribed α inside the range $(0, \alpha_M)$. A refined version of the algorithm allows us to minimize the cross-layer correlations that unavoidably appear for values of α beyond a critical overlap $\alpha_c < \alpha_M$. Finally, we present a very simple example of an SIR epidemic model with information dissemination and use the algorithms to determine the impact of the overlap on the final outbreak size predicted by the model.

PACS numbers: 89.75.-k,89.75.Kd

I. INTRODUCTION

Some contagious processes interact with each other during their propagation, which can occur either through the same route of transmission or through routes that share the same set of nodes but use different types of connections. In the second case, the description of the spread uses the concept of multilayer or multiplex network, namely, a set of nodes (individuals, computers, etc.) connected by qualitatively different types of links corresponding to possible relationships among them (acquaintanceship, friendship, physical contact, social networks, etc), each layer defined by a type of connection. Competitive viruses spreading simultaneously through different routes of transmission over the same host population, or the spread of a pathogen and awareness during an epidemic episode are examples of processes that are better described by means of multilayer networks [1].

In the last years it has been a development of the mathematical formulation of multiplex networks and, also, of more general interconnected networks for which the set of nodes does not need to be the same at each layer [2–4]. Moreover, recent results show the importance of the interrelation between different layers in determining the fate of competitive epidemic processes [1, 5]. In other cases, however, the importance of such an interrelation is not so evident from the analytical results of the epidemic threshold [6, 7], or even seems to be not relevant at all [8].

Only a few papers dealing with competing epidemics over multilayer networks focus on the impact of layer overlap on the epidemic dynamics [5, 9, 10]. In [5], the authors consider a sequential propagation of two epidemics using distinct routes of transmission over a network consisting of two partly overlapped layers. Using bond percolation, it is determined the success of a second epidemic through that part of its route of transmission whose nodes have not been infected by the first epidemic. In [10], the authors develop an analytical approach to deal with simultaneous spread of two interacting viral agents on two-layered networks. In that work, moreover, the respective effects of overlap and correlation of the degrees of nodes in each layer on the epidemic dynamics are considered.

Here the *overlap* α between two (labeled) networks A and B of N nodes is defined as the fraction of links of the union network that are common links of A and B or, equivalently, the probability that a randomly chosen link of the network $A \cup B$ is simultaneously a link of both A and B. In fact α is the *Jaccard index*, a statistic used for comparing the similarity of two sample sets, as defined in [11]. Just to illustrate that this simple statistical parameter can play a critical role in the qualitative response of a two-layer network model, in Section VIII we present a mean-field model for the spread of an infectious agent on one layer (contact layer). The model implicitly assumes an information dissemination on a second layer (notification layer) about the infection status of the nodes which causes a raise in awareness and the adoption of preventive behaviors. As an interesting feature, the overlap coefficient α between the networks embedding the respective routes of transmission is a parameter of the model. This allows us to derive a simple relationship between α and the epidemic threshold. Provided that one wants to perform simulations to validate this (or any) model, a systematic procedure to generate couples of networks

* david.juher@udg.edu
† joan.saldana@udg.edu