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Asymptotic behavior of connecting-nearest-neighbor models for growing networks

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Abstract

This paper deals with the mathematical description of the asymptotic behavior of the solutions of a couple of models for the dynamics of growing networks based on connecting, with a higher probability, nodes that have a neighbor in common. The first model, proposed by A. Vázquez, is nonlinear and, in general, the long-time behavior of the solutions differs from the one predicted by the linear reduction proposed in its original treatment. A second model is specifically derived from the rules defining an *in silico* model also proposed by Vázquez to simulate the growth of a network under the mechanism of connecting nearest neighbors. The two analytical models lead to very different predictions for the configuration of the network that are tested using the simulations of the in silico model.
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Keywords: Growing networks; Continuum approach; Asymptotic behavior of network models

1. Introduction

The study of complex networks started in the 1950s and was motivated by the analysis of the spread of information/infectious diseases through populations with a certain contact structure. This process was, for instance, one of the main motivations for the first percolation model [2] and for the development of the theory of random and biased networks [3–5]. In the 1960s and 1970s the study of social networks received a wave of interest, especially from Milgram's empirical work [6]. This interest was focused on the structure of the network itself and, in particular, on the so-called small-world property, defined by a low average distance ("handshakes") between any pair of persons in the network.

A social network is a set of people with some pattern of interactions (friendship, acquaintances, sexual contacts, business relationships) among them. One of the relevant features of this sort of network is the presence of a high degree of transitivity, i.e., a high probability that one's acquaintances will also be acquainted with each other [7,8]. This fact as well as the presence of "strong" and "weak" ties in the relationships between individuals have been discussed and their consequences have been analyzed for a long time in social network research. At the beginning, such a research pursued the statistical description of networks (average shortest path length, clustering, degree distribution). During the 1980s, much effort was invested in developing geometrical methods in order to represent and compare complex social networks [9].

However, more recently there has been a change in the philosophy of network modeling. In the former models, the goal was to capture correctly the topological aspects of complex networks by constructing graph models based on the classical Erdös–Rényi random graph [5,10]. In contrast to this static approach, since the end of the 1990s, the modeling emphasis has lain on capturing processes governing the growth or evolution of complex networks, the network topology being considered only a by-product of this dynamics [11,12]. In other words, such a modeling approach assumes the idea that a suitable growing mechanism has to lead to a correct prediction of its topology. One such growing mechanism is preferential linking. Introduced in [13], it simply assumes that, in growing networks, the new nodes are attached preferentially to those in

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