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The 3-dimensional cored and logarithm potentials: Periodic orbits

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We study analytically families of periodic orbits for the cored and logarithmic Hamiltonians with 3 degrees of freedom, which are relevant in the analysis of the galactic dynamics. First, after introducing a scale transformation in the coordinates and momenta with a parameter ε, we show that both systems give essentially the same set of equations of motion up to first order in ε. Then the conditions for finding families of periodic orbits, using the averaging theory up to first order in ε, apply equally to both systems in every energy level $H = h > 0$ showing the existence of at least 3 periodic orbits, for ε small enough, and also provides an analytic approximation for the initial conditions of these periodic orbits. We prove that at every positive energy level the cored and logarithmic Hamiltonians with 3 degrees of freedom have at least three periodic solutions. The technique used for proving such a result can be applied to other Hamiltonian systems. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4901126]

I. INTRODUCTION

In this paper, we are interested in 3-degrees Hamiltonian systems of the form

$$H(x, y, z, p_x, p_y, p_z) = \frac{1}{2}\left(p_x^2 + p_y^2 + p_z^2\right) + V(x^2, y^2, z^2),$$

where V a smooth potential with an absolute minimum and a reflection symmetry with respect the three axes. The motivation for the choice of these symmetries becomes from the interest of these potentials in galactic dynamics. In particular, we considered the cored potential

$$V_C = \sqrt{1 + x^2 + y^2 + \frac{z^2}{q^2}} \tag{1}$$

and the logarithm potential

$$V_L = \frac{1}{2}\log\left(1 + x^2 + y^2 + \frac{z^2}{q^2}\right). \tag{2}$$

such potentials in 2-degrees of freedom have been studied by several authors, see, for instance Refs. 1, 3–5, 7–10, 12, and 13

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