Proceedings of the Royal Society of Edinburgh, **143A**, 1291–1302, *2013*

Morphisms and inverse problems for Darboux integrating factors

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(MS received 4 October 2011; accepted 17 August 2012)

Polynomial vector fields which admit a prescribed Darboux integrating factor are quite well understood when the geometry of the underlying curve is non-degenerate. In the general setting, morphisms of the affine plane may remove degeneracies of the curve, and thus allow more structural insight. In the present paper we establish some properties of integrating factors subjected to morphisms, and we discuss in detail one particular class of morphisms related to finite reflection groups. The results indicate that degeneracies for the underlying curve generally impose additional restrictions on vector fields admitting a given integrating factor.

1. Introduction and preliminaries

This paper continues, and to some extent concludes, our work on inverse problems in the Darboux theory of integrability in the affine plane. The inverse problem for Darboux integrating factors is to characterize and determine all polynomial vector fields that admit a prescribed integrating factor. The inverse problem for invariant algebraic curves (which is a part of the former) asks for all polynomial vector fields that admit a given collection of algebraic curves as invariant sets.

Inverse problems in the Darboux theory of integrability are of interest because their solution is necessary to identify and classify the vector fields admitting a Darboux integrating factor. Moreover, such inverse problems have useful applications. For instance, Christopher [1] used an inverse problem to produce polynomial vector fields with algebraic limit cycles, and in [6] solutions of inverse problems were employed to determine vector fields of small degree with a prescribed limit cycle configuration.