ON THE DYNAMICS AND TOPOLOGY OF THE ELLIPTIC RECTILINEAR RESTRICTED 3-BODY PROBLEM

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Abstract. We study a symmetric collinear restricted 3-body problem, where the equal mass primaries perform elliptic collisions, while a third massless body moves in the line between the primaries, during the time between two consecutive elliptic collisions. After desingularizing binary and triple collisions, we prove the existence of a transversal heteroclinic orbit beginning and ending in triple collision. This orbit is the unique homothetic orbit that the problem possess. Finally, we describe the topology of the compact extended phase space.

Key words: 3-body problem, collinear restricted problem, regularization.

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1. Introduction

Restricted 3-body problems in Celestial Mechanics have been considered as simplified models, in view of the complexity of full 3-body problems. They are obtained as a limiting case when one of the masses tends to zero. The first one to be considered was the circular planar restricted 3-body problem [9], modelling, for instance, the motion of an asteroid under the gravitational influence of the Sun and Jupiter. Another important model is the Sitnikov problem [7], in which it was proved for the first time the existence of oscillatory motion in 3-body problems. In both cases the massive bodies or primaries perform periodic motions about each other (circular and elliptical, respectively), while the infinitesimal body moves in the same plane or in the perpendicular line through the center of mass, respectively. By using symbolic dynamics, the existence of periodic and recurrent motions was shown.

Generalizations of the first model, like the elliptical planar (and spatial) restricted 3-body problems have been considered also. The Sitnikov problem belongs to a class of more symmetrical restricted 3-body problems, where the primaries have equal masses. Several generalizations of the Sitnikov problem have been considered lately by Meyer and Wang [6] and Cors and Llibre [1–3], where the



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