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Dynamics of a galactic Hamiltonian system

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We study an even polynomial potential which appears in the study of the galactic dynamics. We prove the existence of four families of periodic orbits in every positive energy level, and we compute an analytic approximation of them. Using such periodic orbits we provide information about the non-integrability of this Hamiltonian system.
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I. INTRODUCTION AND STATEMENTS OF MAIN RESULTS

Galactic dynamics is one of the branches of Astrophysics whose development started only around sixty years ago, when it was possible to have a view of the physical world beyond the integrable and near integrable systems.[7] Even the importance of the analysis of galactic potentials, the global dynamics of galaxies is not a simple question and represents a big challenge for the researches in the field.[6]

To determine interesting properties of the orbital structure of non-integrable potentials is a fundamental topic in galactic dynamics. To know the existence and stability of periodic orbits of low commensurability is important for clarifying some behaviors of the elliptical galaxies. The numerical studies are usually preferred due to the availability of reliable algorithms and powerful machines. On the other hand, in several cases it is useful to have some simple analytic results concerning the relation between the form of the gravitational potential and the main families of orbits exhibited. In this work, we use an analytical technique, the averaging theory, which allows to find periodic orbits of a differential system. We note that this technique is very general and can be applied to many other potentials.

In these last years many articles on galactic dynamics have been focused on models of elliptical galaxies. In most of these models the terms in the potential are of even order, so we have adopted this fact in the Hamiltonian system that we shall analyze. Another important point that appears in these kind of potentials is that the existence of periodic orbits is a useful tool for constructing new and more complicated self consistent models. One way to identify periodic orbits is to localize the central fixed points on the surfaces of constant energy. In [21] the authors studied the localization of periodic orbits and their linear stability for a particular two-component galactic potential. In fact, in our days the study of individual orbits in some galactic potentials is a new branch of galactic dynamics.[8]

The computation of particular orbits in some analytical potentials modeling elliptical galaxies, indicates that relatively small symmetry breaking corrections can increase dramatically the number of stochastic orbits, showing the importance of the study of perturbations of simple models.[9] The class of potentials studied in this paper have not chosen with the aim of modeling some particular galaxies, our objective is to study systems which are generic in their basic properties.

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