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Periodic orbits for the generalized Yang–Mills Hamiltonian system in dimension 6

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Abstract We apply the averaging theory to study a generalized Yang–Mills Hamiltonian system in dimension 6 with six parameters. We provide sufficient conditions on the six parameters of the system which guarantee the existence of continuous families of period orbits parameterized by the energy.

Keywords Periodic orbits · Yang–Mills · Averaging theory

1 Introduction

We study the generalized classical Yang–Mills Hamiltonian system in dimension 6. It consists of a harmonic oscillator plus a homogenous potential of fourth degree with six real parameters a, b, c, d, e, and f.

$$\begin{aligned} H = {} & \frac{1}{2}\left(p_x^2 + p_y^2 + p_z^2 + x^2 + y^2 + z^2\right) \\ & + \frac{1}{4}\Big(ax^4 + 2bx^2y^2 + 2cx^2z^2 \\ & + dy^4 + 2ey^2z^2 + fz^4\Big). \end{aligned} \tag{1}$$

When $z = p_z = 0$ the previous Hamiltonian contains the planar classical Yang–Mills Hamiltonian system. The periodic solutions of this system when $z = p_z = 0$ were studied in [15]. Our aim is to study the periodic solutions in the different energy levels $H = h$ of the Hamiltonian system associated to the Hamiltonian (1).

The mentioned planar Hamiltonian system ($z = p_z = 0$) for $a = 0$ was studied by Contopoulos and co-workers during many years, such a Hamiltonian is now known as the Contopoulos Hamiltonian which describes the perturbed central part of an elliptical or barred galaxy without escapes. For more details see Refs. [5–7]. See also the article of Deprit and Elipe [8] where several periodic orbits and bifurcations are studied for this planar Hamiltonian system. When the quadratic part $(x^2 + y^2)/2 = 0$ and $d = 0$ we obtain the mechanical Yang–Mills Hamiltonian $H = (p_x^2 + p_y^2)/2 + bx^2y^2/2$; where the term x^2y^2 characterizes the Yang–Mills potential, which arises in connection with the classical Yang–Mills field with gage group SU(2) for a homogeneous two-component field, see [11]. Several authors studied quartic homogeneous potentials (without quadratic terms), see for instance Refs [2,3,10]. Moreover, when $b \neq 0$ it is well known that the Hamiltonian of Yang–Mills is non-integrable and strongly chaotic. Others studies and investigations related with generalizations of the mechanical Yang–Mills Hamiltonian have treated quartic terms with three up to five terms in [4,9,13,16]. Maciejewski et al. [16] studied generalized Yang–Mills Hamiltonian systems, which have a quadratic potential plus a homogeneous of fourth degree potential with five parameters, and they proved the existence of connected branches of

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