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PERIODIC ORBITS FOR A GENERALIZED FRIEDMANN-ROBERTSON-WALKER HAMILTONIAN SYSTEM IN DIMENSION 6

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Abstract. A generalized Friedmann-Robertson-Walker Hamiltonian system is studied in dimension 6. The averaging theory is the tool used to provide sufficient conditions on the six parameters of the system which guarantee the existence of continuous families of period orbits parameterized by the energy.

1. **Introduction and statement of the main results.** In astrophysics the study of the dynamic of galaxies progressed considerably thanks to the discovery of important theories coming from mathematical models, see for instance the articles [2], [7], [10], [13].

Calzeta and Hasi (1993) studied the simplified Friedmann-Robertson-Walker Hamiltonian in dimension 4 given by the Hamiltonian $H = (p_y^2 - p_x^2 + y^2 - x^2)/2 + (bx^2y^2)/2$ which is a model for a universe. They proved analytically and numerically the existence of chaotic motion of the Hamiltonian system associated to the above Hamiltonian. Although this model is too simplified to be considered realistically, it is an interesting testing ground for the implications of chaos in cosmology, see for more details [3]. Hawking [4] and Page [8] considered similar models for understanding the relation between the thermodynamic and cosmological arrow of time, in the area of quantum cosmology.

We study the following generalized classical Friedmann-Robertson-Walker Hamiltonian system in dimension 6.

$$H = \frac{1}{2}(p_y^2 + p_z^2 - p_x^2 + y^2 + z^2 - x^2) + \frac{1}{4}(ax^4 + 2bx^2y^2 + 2cx^2z^2 + dy^4 + 2ey^2z^2 + fz^4), \tag{1}$$

Note that this Hamiltonian depends on six parameters a,b,c,d,e and f.

When $z = p_z = 0$ the previous Hamiltonian contains the planar classical Friedmann-Robertson-Walker Hamiltonian system studied by Calzeta and Hasi. Its periodic solutions were studied in [6].

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Key words and phrases. Periodic orbits, Friedmann-Robertson-Walker, averaging theory, family of periodic orbits, periodic orbits parameterized by the energy.

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