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THE FULL PERIODICITY KERNEL OF THE TREFOIL

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1. Introduction and main results.

Let E be a topological space. We shall study some properties of the set of periods for a class of continuous maps from E into itself. We need some notation.

The set of natural numbers, real numbers and complex numbers will be denoted by $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{C}$ respectively. For a map $f : E \to E$ we use the symbol f^n to denote $f \circ f \circ \cdots \circ f$ ($n \in \mathbb{N}$ times), f^0 or «id» denotes the identity map of E. Then, for a point $x \in E$ we define the *orbit* of x, denoted by $\mathrm{Orb}_f(x)$, as the set $\{f^n(x) : n = 0, 1, 2, \ldots\}$. We say x is a *fixed point* of f if $f(x) = x$. We say x is a *periodic point of f of period $k \in \mathbb{N}$* (or simply a *k-point*) if $f^k(x) = x$ and $f^i(x) \neq x$ for $1 \leq i < k$. In this case we say the orbit of x is a *periodic orbit of period k* (or simply a *k-orbit*). Note that if x is a k-point, then $\mathrm{Orb}_f(x)$ has exactly k elements, each of which is a k-point. We denote by $\mathrm{Per}(f)$ the set of periods of all periodic points of f.

A *connected finite regular graph* (or just a *graph* for short) is a pair consisting of a connected Hausdorff space E and a finite subspace V, whose elements are called *vertices*, such that the following conditions hold:

(1) $E \setminus V$ is the disjoint union of a finite number of open subsets $e_1, \ldots, e_k$, called *edges*. Each e_i is homeomorphic to an open interval of the real line.

(2) The boundary, $\mathrm{Cl}(e_i) \setminus e_i$, of the edge e_i consists of two distinct vertices, and the pair $(\mathrm{Cl}(e_i), e_i)$ is homeomorphic to the pair $([0,1],(0,1))$.

If v and e are the number of vertices and edges respectively of E, then *the Euler characteristic of* E, is $\chi(E) = v - e$. A vertex which belongs to

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