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On the full periodicity kernel for one-dimensional maps

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Abstract. Let $\propto$ be the topological space obtained by identifying the points 1 and 2 of the segment $[0, 3]$ to a point. Let ∞ be the topological space obtained by identifying the points 0, 1 and 2 of the segment $[0, 2]$ to a point. An $\propto$ (respectively ∞) map is a continuous self-map of $\propto$ (respectively ∞) having the branching point fixed. Set $E \in \{\propto, \infty\}$. Let f be an E map. We denote by $\mathrm{Per}(f)$ the set of periods of all periodic points of f. The set $K \subset \mathbb{N}$ is the *full periodicity kernel* of E if it satisfies the following two conditions: (1) if f is an E map and $K \subset \mathrm{Per}(f)$, then $\mathrm{Per}(f) = \mathbb{N}$; (2) for each $k \in K$ there exists an E map f such that $\mathrm{Per}(f) = \mathbb{N} \setminus \{k\}$. In this paper we compute the full periodicity kernel of $\propto$ and ∞.

1. *Introduction and main results*

Let E be a topological space. We shall study some properties of the set of periods for a class of continuous maps from E into itself. We need some notation.

The sets of natural numbers, real numbers and complex numbers will be denoted by $\mathbb{N}, \mathbb{R}$ and $\mathbb{C}$ respectively. For a map $f : E \to E$ we use the symbol f^n to denote $f \circ f \circ \cdots \circ f$ ($n \in \mathbb{N}$ times), f^0 denotes the identity map of E. Then, for a point $x \in E$ we define the *orbit* of x, denoted by $\mathrm{Orb}_f(x)$, as the set $\{f^n(x) : n = 0, 1, 2, \ldots\}$. We say x is a *fixed point* of f if $f(x) = x$. We say x is a *periodic point of* f *of period* $k \in \mathbb{N}$ (or simply a k-*point*) if $f^k(x) = x$ and $f^i(x) \neq x$ for $1 \leq i < k$. In this case we say the orbit of x is a *periodic orbit of period* k (or simply a k-*orbit*). Note that if x is a periodic point of period k, then $\mathrm{Orb}_f(x)$ has exactly k elements, each of which is a periodic point of period k. We denote by $\mathrm{Per}(f)$ the set of periods of all periodic points of f.

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