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Diffusion and reaction in the cell glycocalyx and the extracellular matrix

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Abstract Many biologically important macromolecular reactions are assembled and catalyzed at the cell lipid-surface and thus, the extracellular matrix and the glycocalyx layer mediate transfer and exchange of reactants and products between the flowing blood and the catalytic lipid-surface. This paper presents a mathematical model of reaction–diffusion equations that simply describes the transfer process and explores its influence on surface reactivity for a prototypical pathway, the tissue factor (Tf) pathway of blood coagulation. The progressively increasing friction offered by the matrix and glycocalyx to reactants and to the product (coagulation factors X, VIIa and Xa) approaching the reactive surface is simulated and tested by solving the equations numerically with both, monotonically decreasing and constant diffusion profiles. Numerical results show that compared to isotropic transfer media, the anisotropic structure of the matrix and glycocalyx sharply decreases overall reaction rates and significantly increases the mean transit time of reactants; this implies that the anisotropy modifies the distribution of reactants. Results also show that the diffusional transfer, whether isotropic or anisotropic, influences reaction rates according to the order at which the reactants arrive at the boundary. Faster rates are observed when at

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