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Linear estimate for the number of zeros of Abelian integrals for quadratic isochronous centres

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Abstract. The main objective of this paper is to provide an explicit and fairly accurate upper bound for the number of zeros of Abelian integrals defined by quadratic isochronous centres when we perturb them inside the class of all polynomial systems of degree n.

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1. Introduction and statement of the main results

In the qualitative theory of real planar differential systems the main open problem is the determination of limit cycles. Probably the more usual way to obtain limit cycles is perturbing the periodic orbits of a centre, in such a way that the centre is destroyed in the perturbed system which can exhibit limit cycles.

There are several methods for studying bifurcated limit cycles from a centre. The majority of the methods are based either on the Poincaré return map or Abelian integrals (see, for instance, Pontryagin [22], or more recently [1]). Recently, some other methods have been presented, ones based on the inverse integrating factor (see [5]) and others based on the reduction of the problem to a one-dimensional differential equation (see [12, 14]). In general, these methods are difficult to apply to study limit cycles that bifurcate from the periodic orbits of a centre when the system is integrable but not Hamiltonian. As far as we know few papers study the non-Hamiltonian centres, see for instance [2, 3, 6, 12, 14, 17].

By definition a *polynomial system* is a differential system of the form

$$\frac{\mathrm{d}x}{\mathrm{d}t} = P(x, y) \qquad \frac{\mathrm{d}y}{\mathrm{d}t} = Q(x, y) \tag{1}$$

where P and Q are polynomials with real coefficients. We say that $n = \max\{\deg P, \deg Q\}$ is the *degree* of the polynomial system.

In what follows systems (1) of degree two are called *quadratic systems*. Over the last 30 years quadratic systems have been studied intensively, and more than 1000 papers have been published about them (see, for instance, the bibliographical survey by Reyn [23]). Many authors have studied the limit cycles which bifurcate from periodic orbits of a centre for a quadratic system (see, for instance, [8, 16, 20, 21, 24, 26]).

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