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# ON THE LIMIT CYCLES OF POLYNOMIAL DIFFERENTIAL SYSTEMS WITH HOMOGENEOUS NONLINEARITIES

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*Abstract* We consider three classes of polynomial differential equations of the form $\dot{x} = -y + P_n(x, y)$, $\dot{y} = x + Q_n(x, y)$, where $P_n$ and $Q_n$ are homogeneous polynomials of degree $n$, having a non-Hamiltonian centre at the origin. By using a method different from the classical ones, we study the limit cycles that bifurcate from the periodic orbits of such centres when we perturb them inside the class of all polynomial differential systems of the above form. A more detailed study is made for the particular cases of degree $n = 2$ and $n = 3$.

*Keywords:* limit cycles; centres; bifurcation

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## 1. Introduction

The main problem in the qualitative theory of real planar differential systems is the determination of limit cycles. Limit cycles of planar vector fields were defined by Poincaré [**30**] and started to be studied intensively at the end of the 1920s by van der Pol [**34**], Liénard [**24**] and Andronov [**1**].

One of the classical ways to produce limit cycles is by perturbing a system which has a centre in such a way that limit cycles bifurcate in the perturbed system from some of the periodic orbits of the unperturbed system (see, for example, [**31**]).

In this paper we consider three subclasses of real planar polynomial differential systems of the form

$$\dot{x} = -y + P_n(x, y), \qquad \dot{y} = x + Q_n(x, y), \tag{1.1}$$

where $P_n$ and $Q_n$ are homogeneous polynomials of degree $n$ having a centre at the origin, and we study the limit cycles which bifurcate from their periodic orbits when we perturb such subclasses inside the class of all system (1.1) (see Theorem 2.3). In the future we want to study the perturbation of the centres of system (1.1) with $n = 2$ inside the class of all polynomial systems of arbitrary degree.