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Linear Estimate of the Number of Zeros of Abelian Integrals for a Class of Integrable non-Hamiltonian Systems

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Abstract

We consider the class of all polynomial systems having a conic center (i.e. all periodic orbits round the center are conics). It is proved that all such systems have an isochronous center, and up to affine transformations of the coordinates, each such system coincides with linear isochronous center (S_0), quadratic isochronous centers (S_1),(S_2) due to Loud's classification [1] or a special kind of a cubic isochronous system (S_*). Moreover, it is proved that the upper bounds of the number of zeros of Abelian integrals of (S_1), (S_2) and (S_*) in case of time reversible, are linearly dependent on n, when such systems are perturbed inside the class of all polynomial systems of degree n.

1 Introduction

We consider planar polynomial integrable systems with polynomial perturbations:

$$\begin{aligned} \frac{dx}{dt} &= X(x,\ y) + \varepsilon P(x,y), \\ \frac{dy}{dt} &= Y(x,\ y) + \varepsilon Q(x,y), \end{aligned} \qquad (1.1)_\varepsilon$$

where $0 < |\varepsilon| \ll 1, \max(\deg X, \deg Y) = m, \max(\deg P, \deg Q) = n$. It is assumed that for $\varepsilon = 0$, system $(1.1)_0$ has a first integral $H(x,y)$ with integrating factor $M(x,y)$, i.e.

$$X(x,y) = -\frac{1}{M}\frac{\partial H}{\partial y}, \quad Y(x,y) = \frac{1}{M}\frac{\partial H}{\partial x}.$$

Let $\Sigma \subset \mathbf{R}$ be the maximal connected open interval of existence of a continuous family of ovals Γ_h of the real curves $H(x,y) = h \in \mathbf{R}$. The first approximation of the Poincaré return map of $(1.1)_\varepsilon$ for Γ_h with respect to the small parameter ε is defined by the complete Abelian integral

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