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Polynomial Systems: a Lower Bound for the Weakened 16th Hilbert Problem

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In this paper we provide the greatest lower bound about the number of (non–infinitesimal) limit cycles surrounding a unique singular point for a planar polynomial differential system of arbitrary degree.

We prove that for m and n odd the maximum number $b_{m,n}$ of isolated zeros (taking into account their multiplicity) of the Abelian integral $I(h) = \int_{H(x,y)=h} y\bar{Q}(x,y)dx$, where $H(x,y) = \frac{1}{2}y^2 + \frac{1}{m+1}x^{m+1}$, and $\bar{Q}$ and arbitrary polynomial of degree at most $n-1$ is

$$\frac{(n+1)(n+3)}{8} - 1 \quad \text{if} \quad n \le m, \qquad \frac{(m+1)(2n-m+3)}{8} - 1 \quad \text{if} \quad n \ge m.$$

Moreover, there are perturbations of the Hamiltonian system $\dot{x} = -\partial H/\partial y$, $\dot{y} = \partial H/\partial x$, such that the indicated maximum number $b_{m,n}$ of continuous families of limit cycles can be made to emerge from a corresponding number of arbitrarily prescribed periodic orbits within the period annulus of the center. Consequently,

$$b_{m,n} \le N(m,n) \le H_{\max\{m,n\}}.$$

This result provides the greatest lower bound about the number of (non–infinitesimal) limit cycles surrounding a unique singular point for a planar polynomial differential system of arbitrary degree $m = n$.