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NEW FAMILIES OF CENTERS AND LIMIT CYCLES FOR POLYNOMIAL DIFFERENTIAL SYSTEMS WITH HOMOGENEOUS NONLINEARITIES *†

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Dedicated to Professor Ye Yanqian on the occasion of his 80th birthday

Abstract

We consider the class of polynomial differential equations $\dot{x} = -y + P_n(x, y)$, $\dot{y} = x + Q_n(x, y)$, where P_n and Q_n are homogeneous polynomials of degree n. Inside this class we identify a new subclass of systems having a center at the origin. We show that this subclass contains at least two subfamilies of isochronous centers. By using a method different from the classical ones, we study the limit cycles that bifurcate from the periodic orbits of such centers when we perturb them inside the class of all polynomial differential systems of the above form. In particular, we present a function whose simple zeros correspond to the limit cycles which bifurcate from the periodic orbits of Hamiltonian systems.

Keywords limit cycles, centers, bifurcation
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1 Introduction

Two of the main problem in the qualitative theory of real planar differential systems are the determination of centers and of limit cycles. Limit cycles of planar vector fields were defined by Poincaré [44], and started to be studied intensively at the end of 1920s by van der Pol [45], Liénard [34] and Andronov [1]. A *limit cycle* is a periodic orbit of the planar differential system isolated in the set of all periodic orbits.

One of the classical ways to produce limit cycles is by perturbing a system which has a center, in such a way that limit cycles bifurcate in the perturbed system from

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