

A Unified Study on the Cyclicity of Period Annulus of the Reversible Quadratic Hamiltonian Systems*

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The cyclicity of the period annulus of reversible quadratic Hamiltonian systems under quadratic perturbations was studied by several authors for different cases by using different methods. In this paper, we study this problem in a unified way.

KEY WORDS: Cyclicity of period annulus; reversible quadratic Hamiltonian systems.

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1. INTRODUCTION

Following [11,17] the quadratic centers are classified into four classes: Hamiltonian (Q_3^H), reversible (Q_3^R), Lotka–Volterra (Q_3^{LV}) and codimension four (Q_4). It was pointed out by Iliev in [9] that if $X_H \in Q_3^H \cap Q_3^R$, then except one case (the Bogdanov–Takens system, which has been extensively studied in many papers, see [1,9,12,13]), the cubic Hamiltonian $H(x, y)$ can be transformed into the following one-parameter family

$$H_A(x, y) = x[3(A - 2) - 3(A - 1)x + Ax^2 + y^2], \quad A \in (-\infty, \infty). \quad (1.1)$$

The phase portraits of the corresponding vector fields

$$X_{H_A} = \frac{\partial H_A}{\partial y} \frac{\partial}{\partial x} - \frac{\partial H_A}{\partial x} \frac{\partial}{\partial y}$$

*Dedicated to Professor Shui-Nee Chow on the occasion of his 60th birthday.

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