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Uniqueness of limit cycles for Liénard differential equations of degree four

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ABSTRACT

We prove that any classical Liénard differential equation of degree four has at most one limit cycle, and the limit cycle is hyperbolic if it exists. This result gives a positive answer to the conjecture by A. Lins, W. de Melo and C.C. Pugh (1977) [4] about the number of limit cycles for polynomial Liénard differential equations for $n = 4$.

1. Introduction

The study of Liénard differential equations has a long history and a lot of results were obtained, see [8] for example. A classical polynomial Liénard differential equation can be written as a planar system

$$\begin{aligned} \dot{x} &= y - F(x), \\ \dot{y} &= -x, \end{aligned} \tag{1.1}$$

where $F(x)$ is a polynomial of degree n. In 1977 A. Lins, W. de Melo and C.C. Pugh conjectured in [4] that the classical Liénard differential equation of degree $n \geqslant 3$ has at most $\left[\frac{n-1}{2}\right]$ limit cycles, where $\left[\frac{n-1}{2}\right]$ means the largest integer less than or equal to $\frac{n-1}{2}$. They also proved that the conjecture is true for $n = 3$. In 2007 F. Dumortier, D. Panazzolo and R. Roussarie [3] gave a counterexample to this

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