CUBIC HOMOGENEOUS POLYNOMIAL CENTERS

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Abstract. First, doing a combination of analytical and algebraic computations, we determine by first time an explicit normal form depending only on three parameters for all cubic homogeneous polynomial differential systems having a center.

After using the averaging method of first order we show that we can obtain at most 1 limit cycle bifurcating from the periodic orbits of the mentioned centers when they are perturbed inside the class of all cubic polynomial differential systems. Moreover, there are examples with 1 limit cycles.

1. Introduction and statement of the main results

We consider *polynomial differential systems* in $\mathbb{R}^2$ of the form

$$\dot{x} = P(x, y), \qquad \dot{y} = Q(x, y), \tag{1}$$

where P and Q are real polynomials in the variables x and y. We say that this system has *degree* m, if m is the maximum of the degrees of P and Q.

In his address to the International Congress of Mathematicians in Paris in 1900, Hilbert [7] asked for the maximum number of limit cycles which real polynomial differential systems of degree m could have, this problem is known as the 16th *Hilbert problem*, for more details about it see the surveys [8, 9].

Since the 16th Hilbert problem is too much difficult Arnold, in [1], stated the *weakened* 16*th Hilbert problem*, one version of this problem is to determine an upper bound for the number of limit cycles which can bifurcate from the periodic orbits of a polynomial Hamiltonian center when it is perturbed inside a class of polynomial differential systems, see for instante [3, 4, 6].

We recall that a singular point $p \in \mathbb{R}^2$ of a differential system (1) is a *center* if there is a neighborhood U of p such that $U \setminus \{p\}$ is filled of periodic orbits of (1). The *period annulus* of a center is the region fulfilled by all the periodic orbits surrounding the center. We say that a center located at the origin is *global* if its period annulus is $\mathbb{R}^2 \setminus \{0\}$.

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