Chaos, Solitons and Fractals 120 (2019) 149–157

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Frontiers

Phase portraits of continuous piecewise linear Liénard differential systems with three zones

Shimin Li[a,*], Jaume Llibre[b]

[a] School of Mathematics and Statistics, Guangdong University of Finance and Economics, Guangzhou 510320, PR China
[b] Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain

ARTICLE INFO

Article history:
Received 24 April 2018
Revised 7 December 2018
Accepted 12 December 2018

MSC:
34A36
34C07
37G05

Keywords:
Limit cycle
Piecewise linear differential system
Phase portrait
Liénard system

ABSTRACT

Phase portraits are an invaluable tool in studying differential systems. Most of known results about global phase portraits are related to the smooth differential systems.

This paper deals with a class of planar continuous piecewise linear Liénard differential systems with three zones separated by two vertical lines without symmetry. We provide the topological classification of the phase portraits in the Poincaré disc for systems having a unique singular point located in the middle zone.

1. Introduction and main results

In recent years there is a growing interest in the analysis of planar continuous piecewise linear Liénard differential systems of the form:

$$\frac{dx}{dt} = y - f(x), \qquad \frac{dy}{dt} = a - g(x), \tag{1}$$

where

$$f(x) = \begin{cases} k_1(x-1)+k_2 & \text{if } x > 1, \\ k_2 x & \text{if } -1 \leqslant x \leqslant 1, \\ k_3(x+1)-k_2 & \text{if } x < -1, \end{cases} \tag{2}$$

and

$$g(x) = \begin{cases} l_1(x-1)+l_2 & \text{if } x > 1, \\ l_2 x & \text{if } -1 \leqslant x \leqslant 1, \\ l_3(x+1)-l_2 & \text{if } x < -1. \end{cases} \tag{3}$$

Systems (1) satisfying (2) and (3) have been studied extensively, see for instance [1–7]. For the symmetrical cases, that is $a = 0$, $k_1 = k_3$ and $l_1 = l_3$, see [9] for the bifurcation sets and existence of the limit cycles, see [10] for the amplitude and period of the limit cycles, see [18] for the global phase portraits and bifurcation diagrams. While for the non-symmetrical case, the analysis of systems (1) satisfying (2) and (3) become more complicated. In [15] the authors considered the existence and uniqueness of limit cycles for the case $k_1 > 0$, $k_1 < 0$, $k_3 > 0$. In [16] the authors studied the uniqueness and non-uniqueness of limit cycles for the more general cases, they showed that systems (1) satisfying (2) and (3) can have at leat two limit cycles for the cases either $k_1 > 0$, $k_2 < 0$, $k_3 < 0$, or $k_1 < 0$, $k_2 < 0$, $k_3 > 0$. In [7] the authors found that systems (1) satisfying (2) and (3) can exist jump bifurcations when $l_2 = 0$. In [26] the authors investigated the boundary equilibrium bifurcations of systems (1) satisfying (2) and (3).

It is worth to note that if either $k_1 = k_2, l_1 = l_2$, or $k_2 = k_3, l_2 = l_3$, then systems (1) satisfying (2) and (3) become continuous piecewise linear Liénard differential systems with two zones. In 1990 Lum and Chua [20,21] conjectured that a continuous piecewise linear differential system with two zones separated by a straight line has at most one limit cycle. In 1998 Freire et al. [8] gave a positive answer to this conjecture. Note that this conjecture cannot be extended to discontinuous piecewise linear differential systems see [1,13], and to continuous piecewise linear differential systems with non-regular separation line see [3,17]. There are several recent works related to the number of limit cycles for piecewise smooth integrable systems, see for instance [12,27,28].

* Corresponding author.
E-mail addresses: lism1983@126.com (S. Li), jllibre@mat.uab.cat (J. Llibre).

https://doi.org/10.1016/j.chaos.2018.12.037