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ON THE LIMIT CYCLES OF PLANAR DISCONTINUOUS PIECEWISE LINEAR DIFFERENTIAL SYSTEMS WITH A UNIQUE EQUILIBRIUM

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ABSTRACT. This paper deals with planar discontinuous piecewise linear differential systems with two zones separated by a vertical straight line x = k. We assume that the left linear differential system (x < k) and the right linear differential system (x > k) share the same equilibrium, which is located at the origin O(0,0) without loss of generality.

Our results show that if k = 0, that is when the unique equilibrium O(0,0) is located on the line of discontinuity, then the discontinuous piecewise linear differential systems have no crossing limit cycles. While for the case $k \neq 0$ we provide lower and upper bounds for the number of limit cycles of these planar discontinuous piecewise linear differential systems depending on the type of their linear differential systems, i.e. if those systems have foci, centers, saddles or nodes, see Table 2.

1. Introduction and statement of the main results. Planar piecewise linear differential systems are the natural extension of linear differential systems in order to investigate nonlinear dynamics. It is obvious that this class of piecewise linear differential systems with two zones separated by a straight line is the simplest class of these piecewise differential systems. Without loss of generality we can assume that the separation straight line is x = k, then we have

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{cases} \begin{pmatrix} a_{1,1}^- & a_{1,2}^- \\ a_{2,1}^- & a_{2,2}^- \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} b_1^- \\ b_2^- \end{pmatrix} & \text{if } x < k, \\ \begin{pmatrix} a_{1,1}^+ & a_{1,2}^+ \\ a_{2,1}^+ & a_{2,2}^+ \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} b_1^+ \\ b_2^+ \end{pmatrix} & \text{if } x > k, \end{cases}$$
(1)

where the dot denote the derivative with respect to t. We call the linear differential system of (1) defined in x < k (resp. x > k) the left (resp. right) linear differential system.

In 1990 Lum and Chua [22] conjectured that a continuous piecewise linear differential systems (1) has at most one limit cycle. This conjecture has been solved

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