

PLANAR ANALYTIC VECTOR FIELDS WITH GENERALIZED RATIONAL FIRST INTEGRALS

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ABSTRACT. – The main purpose of this paper is to characterize a germ of planar holomorphic vector field at an elementary singular point having a generalized rational first integral. Our results generalize a result due to Poincaré on a necessary condition of the existence of a rational first integral for planar polynomial systems. As two applications of our main result, we give the necessary and sufficient conditions on the existence of rational first integral for planar quadratic systems having either a weak nondegenerate singular point, or a degenerate elementary singular point. © 2001 Éditions scientifiques et médicales Elsevier SAS

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1. Introduction and statement of the main results

A vector field defined in a domain of $\mathbb{C}^n$ is said to be *analytic* or *holomorphic* if its components are holomorphic functions. A holomorphic

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