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Local first integrals of differential systems and diffeomorphisms

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Abstract. In this paper using theory of linear operators and normal forms we generalize a result of Poincaré [11] about the non–existence of local first integrals for systems of differential equations in a neighbourhood of a singular point. As an application of the generalized result, and under more weak conditions we obtain a result of Furta [8] about local first integrals of semi–quasi–homogeneous systems. Moreover, for diffeomorphisms and periodic differential systems we give definitions of their first integrals, and generalize the previous results about systems of differential equations to diffeomorphisms in a neighbourhood of a fixed point and to periodic differential systems in a neighbourhood of a constant solution.

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1. Introduction and statement of the main results

Investigation of first integrals for systems of differential equations is classical work. In recent years there are many authors to look for first integrals or to prove non–integrability of autonomous differential systems (see for instance, [3], [5], [7], [8], [9] and [12]). In the present paper by using theory of normal forms we give a necessary and sufficient conditions for a class of autonomous differential systems to have local (formal) first integrals, which generalize a result of Poincaré [11] and one of Furta [8]. Moreover, we extend these results to local first integrals of diffeomorphisms, and of periodic differential systems.

We consider the following autonomous differential system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}), \qquad \mathbf{x} = (x_1, \cdots, x_n) \in \mathbb{C}^n, \tag{1}$$

where $\mathbf{f}$ is a vector–valued function of dimension n satisfying $\mathbf{f}(\mathbf{0}) = \mathbf{0}$, the dot denotes the derivative with respect to time variable t. As usual, $\mathbb{C}$ denotes the complex field.

Let $U \subset \mathbb{C}^n$ be an open connected subset. A non–constant analytic function $H : U \to \mathbb{C}$ is called an *analytic first integral* of system (1) in U if and only if