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Parallelization of the Lyapunov constants and cyclicity for centers of planar polynomial vector fields

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Abstract

Christopher in 2006 proved that under some assumptions the linear parts of the Lyapunov constants with respect to the parameters give the cyclicity of an elementary center. This paper is devoted to establish a new approach, namely parallelization, to compute the linear parts of the Lyapunov constants. More concretely, it is shown that parallelization computes these linear parts in a shorter quantity of time than other traditional mechanisms.

To show the power of this approach, we study the cyclicity of the holomorphic center $\dot{z} = iz + z^2 + z^3 + \cdots + z^n$ under general polynomial perturbations of degree n, for $n \leq 13$. We also exhibit that, from the point of view of computation, among the Hamiltonian, time-reversible, and Darboux centers, the holomorphic center is the best candidate to obtain high cyclicity examples of any degree. For $n = 4, 5, \ldots, 13$, we prove that the cyclicity of the holomorphic center is at least $n^2 + n - 2$. This result gives the highest lower bound for $M(6), M(7), \ldots, M(13)$ among the existing results, where $M(n)$ is the maximum number of limit cycles bifurcating from an elementary monodromic singularity of polynomial systems of degree n. As a direct corollary we also obtain the highest lower bound for the Hilbert numbers $H(6) \geq 40$, $H(8) \geq 70$, and $H(10) \geq 108$, because until now the best result was $H(6) \geq 39$, $H(8) \geq 67$, and $H(10) \geq 100$.

MSC: 34C23; 34C25; 34C07

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