LIMIT CYCLES FOR A CLASS OF CONTINUOUS PIECEWISE LINEAR DIFFERENTIAL SYSTEMS WITH THREE ZONES

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In this paper we consider a class of planar continuous piecewise linear vector fields with three zones. Using the Poincaré map we show that these systems admit always a unique limit cycle, which is hyperbolic.

1. Introduction

Due to the encouraging increase in their applications, control theory [Lefschetz, 1965] and [Narendra et al., 1973], design of electric circuits [Chua et al., 1990], neurobiology [FitzHugh, 1961] and [Nagumo et al., 1962] piecewise linear differential systems were studied early from the point of view of qualitative theory of ordinary differential equations [Andronov et al., 1966]. Nowadays, a lot of papers are being devoted to these differential systems.

On the other hand, starting from linear theory, in order to capture nonlinear phenomena, a natural step is to consider piecewise linear systems. As local linearizations are widely used to study local behavior, global linearizations (achieved quite naturally by working with models which are piecewise linear) can help to understand the richness of complex phenomena observed in the nonlinear world.

The study of piecewise linear systems can be a difficult task that is not within the scope of traditional nonlinear systems analysis techniques. In particular, a sound bifurcation theory is lacking for such systems due to their nonsmooth character.

In this paper we study the existence of limit cycles for the class of continuous piecewise linear differential systems

$$\mathbf{x}' = X(\mathbf{x}), \tag{1}$$

where $\mathbf{x} = (x, y) \in \mathbb{R}^2$, and X is a continuous piecewise linear vector field. We will consider the following situation, that we will name the three-zone case. We have two parallel straight lines L_- and L_+ symmetric with respect to the origin dividing the phase plane in three closed regions: R_-, R_o and R_+ with $(0, 0) \in R_o$ and the regions R_- and R_+ have as boundary the straight lines L_- and L_+ respectively. We will denote by X_- the vector field X restrict to R_-, by X_o the vector field X restricted