J. Phys. A: Math. Theor. 44 (2011) 365201 (15pp)

doi:10.1088/1751-8113/44/36/365201

Global dynamics of the Rössler system with conserved quantities

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Received 4 May 2011, in final form 20 July 2011 Published 16 August 2011 Online at stacks.iop.org/JPhysA/44/365201

Abstract

The subject of this paper concerns with a class of Rössler systems that admits conserved quantities. For this class of systems a complete description of the global dynamics in the Poincaré sphere is provided.

PACS number: 05.45.-a Mathematics Subject Classification: 34A36, 34A60, 34C25, 34C37

1. Introduction

In general, it is not easy to detect when a differential system has some kind of integrability. This question can cover up an entire book, see for instance [9]. The integrability of a differential system exhibits some recognizable features such as the existence of conserved quantities, or the presence of algebraic invariants, or the ability to give explicit solutions, see for instance [3].

In this paper, we work with the notion of integrability associated with conserved quantities, either with the existence of first integrals or conserved quantities independent of the time, or with the existence of invariants or conserved quantities depending on the time. Clearly, when a system presents first integrals or invariants, they strongly help to understand the dynamics of the system.

In this paper, we study the Rössler system [6]. This system is a three-dimensional system given by

$$\dot{x} = -y - z = P_1(x, y, z), \dot{y} = x + ay = P_2(x, y, z), \dot{z} = b - cz + xz = P_3(x, y, z),$$
(1)

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where a, b, c are real parameters and the dot denotes the derivative with respect to the independent variable t that we call the time.

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