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Integrability of the Rucklidge system

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Abstract We study the Darboux and the analytic integrability of the Rucklidge system.

Keywords Analytic first integrals · Darboux first integrals · Darboux polynomials · Exponential factors

1 Introduction and statement of the main results

The Rucklidge system (see [9]):

. 1

$$x = -ax + by - yz,$$

$$\dot{y} = x,$$

$$\dot{z} = -z + y^{2},$$
(1)

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Departamento de Matemática, Instituto Superior Técnico, Universidade Técnica de Lisboa, Av. Rovisco Pais 1049–001, Lisboa, Portugal e-mail:cvalls@math.ist.utl.pt is a famous model (see for instance [9]), where $x, y, z \in \mathbb{R}^3$ are the state variables, (a, b) are real parameters and the dot denotes the derivative with respect to the time t. This model considers the problem of two-dimensional convection in a horizontal layer of Boussinesq fluid with lateral constants. It provides an accurate description of convection in the parameter regime where the chaotic solutions appear. Despite its simplicity it has a reach local dynamical behavior as chaotic attractors for some values of the parameters a and b (for example when a = 2, b = 6.7 or a = -0.1 and b = -1) and has been widely analyzed (see for instance [10,11] and the references therein).

We note that system (1) is a family of quadratic systems in a three-dimensional space. Quadratic systems in \mathbb{R}^3 are the simplest systems after the linear ones. Examples of such systems are the well-known Lorenz system, Rössler system, Rikitake system, among others. These have been investigated in the last decades from different dynamical points of view. Despite their simplicity, quadratic systems are not completely understood from the view point of the integrability, see for instance [8].

The aim of this paper is to study the existence of Darboux and analytic first integrals of system (1). We recall that a *first integral of Darboux type* is a first integral H which is a function of Darboux type (see below (2) for a precise definition). The study of the integrability is a classical problem in the theory of differential equations.