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HETEROCLINIC ORBITS FOR A CLASS OF HAMILTONIAN SYSTEMS ON RIEMANNIAN MANIFOLDS

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Abstract. Let $\mathcal{M}$ be a smooth Riemannian manifold with the metric (g_{ij}) of dimension n, and let $H = \frac{1}{2}g^{ij}(q)p_ip_j + V(t,q)$ be a smooth Hamiltonian on $\mathcal{M}$, where (g^{ij}) is the inverse matrix of (g_{ij}). Under suitable assumptions we prove the existence of heteroclinic orbits of the induced Hamiltonian systems.

1. **Introduction and statement of the main results.** The existence of homoclinic and heteroclinic orbits for Hamiltonian systems by using the variational methods and critical point theory has been studied by many authors (see for instance, [4]–[6], [10, 11], [14]–[18] and [20]). We must say that Rabinowitz has given fundamental contributions to this field. Our present work is motivated by [14] and [11].

In [14] Rabinowitz studied the autonomous second order Hamiltonian system

$$\ddot{q} + V_q(q) = 0, \qquad q = (q_1, \dots, q_n) \in \mathbb{R}^n, \tag{1}$$

with the function $V : \mathbb{R}^n \to \mathbb{R}$ satisfying the assumptions

(R_1) $V \in C^1(\mathbb{R}^n, \mathbb{R})$, and $V(q) \le 0$ for all $q \in \mathbb{R}^n$.
(R_2) V is periodic in q_i with a period T_i, $1 \le i \le n$.
(R_3) The set $\mathcal{U} = \{y \in \mathbb{R}^n;\, V(y) = 0\}$ consists only of isolated points.

Then for every $x \in \mathcal{U}$, there exist at least two heteroclinic orbits of (1) joining x to $\mathcal{U} \setminus \{x\}$. Moreover, at least one of these orbits emanates from x and at least one terminates at x. As mentioned in [11] Rabinowitz's proof strongly depends on the fact that the system is autonomous.

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