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On the early epidemic dynamics for pairwise models

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HIGHLIGHTS

- The early dynamics of SIS, SIR and SEIR pairwise models with rewiring are studied.
- R_0 is obtained from the early dynamics of the number of susceptibles per infective.
- The analytical relationship between $\mathcal{R}_0$ and the initial growth rate is derived.

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ABSTRACT

The relationship between the basic reproduction number $\mathcal{R}_0$ and the exponential growth rate, specific to pair approximation models, is derived for the SIS, SIR and SEIR deterministic models without demography. These models are extended by including a random rewiring of susceptible individuals from infectious (and exposed) neighbours. The derived relationship between the exponential growth rate and $\mathcal{R}_0$ appears as formally consistent with those derived from homogeneous mixing models, enabling us to measure the transmission potential using the early growth rate of cases. On the other hand, the algebraic expression of $\mathcal{R}_0$ for the SEIR pairwise model shows that its value is affected by the average duration of the latent period, in contrast to what happens for the homogeneous mixing SEIR model. Numerical simulations on complex contact networks are performed to check the analytical assumptions and predictions.

1. Introduction

The success of invasion processes governed by short-range interactions crucially depends on the early development of local densities around the invading individuals, which constitute the so-called invading clusters (Bauch, 2005). These local spatial patterns of individual states define the environmental conditions experienced by the invaders, so determining their final fate. Invading clusters have been observed in lattice models in Ecology (Garcia-Domingo and Saldaña, 2011; Harada and Iwasa, 1994; Harada et al., 1995) as well as in simple epidemic network models (Bauch, 2005; Juher et al., 2013; Keeling, 1999) where invaders correspond to infectious individuals introduced into a totally susceptible population. In both settings, pairwise models offer a useful approach to the study of their dynamics. In such models, local densities are described by either the conditional probabilities that an invader has a neighbour in a particular state (Harada and Iwasa, 1994; Harada et al., 1995), or the mean fraction of neighbours of a given type around an infectious individual (Bauch, 2005; Keeling, 1999; Trapman, 2007), or by the average number of neighbours of a given type per infectious individual (Juher et al., 2013). In Bauch (2005), Keeling (1999) and Trapman (2007) correlations among infection states are defined in terms of local densities and are used to describe the early development of spatial patterns.

The early dynamics of local densities around infectious individuals have been studied from their limit (Bauch, 2005; Juher et al., 2013; Keeling, 1999) for the susceptible–infected–susceptible (SIS) and susceptible–infectious–recovered (SIR) models. These equations are derived by assuming that the expected number of susceptible individuals, $[S]$, and of susceptible–susceptible links, $[SS]$, are approximated to the total number of individuals (nodes), N, and to the total number of links (non-ordered pairs), L, respectively. Under these assumptions, the expected number of susceptible–infectious pairs (infectious links), $[SI]$, infectious–recovered pairs, $[IR]$, and infectious–infectious pairs, $[II]$, becomes zero. However, this behaviour is not necessarily true for the local densities $[SI]/[I]$, $[RI]/[I]$, and $2[II]/[I]$, i.e., the mean number of susceptible, recovered, and infectious individuals around an infectious individual, respectively. Conversely, the so-called mean-field models, like the deterministic SIS, SIR, and SEIR (E for Exposed) models or unstructured

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