Contemporary Mathematics
Volume 152, 1993

Lefschetz numbers for periodic points

JAUME LLIBRE

ABSTRACT. We develop a modified Lefschetz number for analysing if a given period belongs to the set of periods of a self-map. We apply it to study of the periodic points of some classes of maps: transversal maps on compact manifolds, Lie group endomorphisms, torus maps and transversal graph maps.

1. Introduction and statement of the main results

We develop in this paper a modified Lefschetz number for analysing if a given period belongs to the set of periods of a self-map. Essentially we work with the Lefschetz numbers for periodic points instead of the usual Lefschetz number for fixed points. As these techniques are homological they are computable and they apply equally to all maps in an appropriate homotopy class. The modified Lefschetz numbers are applied to study the set of periods of several classes of self-maps.

For simplicity of exposition all spaces considered here will be compact manifolds or finite graphs, and thus they admit an index theory (see, for instance **[6]**).

Let $f : X \to X$ be a continuous map. A *fixed point* of f is a point x of X such that $f(x) = x$. Denote the totality of fixed points by $\text{Fix}(f)$. The point $x \in X$ is *periodic with period* m if $x \in \text{Fix}(f^m)$ but $x \notin \text{Fix}(f^k)$ for all $k = 1, \dots, m-1$. Let $\text{Per}(f)$ denote the set of all periods of periodic points of f.

Let M be a compact manifold of dimension n. A continuous map $f : M \to M$ induces endomorphisms $f_{*k} : H_k(M; \mathbb{Q}) \to H_k(M; \mathbb{Q})$ (for $k = 0, 1, \dots, n$) on

---

1991 *Mathematics Subject Classification.* Primary 58F20.

The author was supported in part by DGICYT Grant PB90-0695.

This paper is in final form and no version of it will be submitted for publication elsewhere.