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SOME CONJECTURES ON PLANAR POLYNOMIAL VECTOR FIELDS AND STRAIGHT LINES [1]

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Abstract. In this note we present some partial results on the Invariant Straight Line Conjecture: Let P and Q be two real polynomials in the variables x and y such that degree of P^2+Q^2 is $2n>2$. Assume that the polynomial differential system $x'=P(x,y)$, $y'=Q(x,y)$, has finitely many invariant straight lines. Then it has at most $2n+1$ (respectively $2n+2$) invariant straight lines if n is even (respectively odd). Furthermore we introduce two new conjectures, one geometric and another algebraic, which imply the Invariant Straight Line Conjecture.

Let P and Q be two real polynomials in the real variables x and y. We say that the polynomial differential system

$$x' = P(x,y)\,, \quad y' = Q(x,y)\,, \tag{1}$$

has *degree* n if the degree of the polynomial P^2+Q^2 is $2n$.

Studies of polynomial differential systems were carried out by Poincaré in [2] and [3]. The algebraic feature of polynomial differential systems renders natural certain questions and problems of an algebraic or an algebro–geometric nature as the following two. Recognize when system (1) has invariant algebraic curves, or is algebraically integrable? See the interesting survey of Schlomiuk [4] on these questions. This paper is about the first question.

The straight line $ax+by+c=0$ is invariant for the flow of system (1), and we call it an *invariant straight line* of system (1) if

[1] This paper is a summary of [1].