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ON THE ALGEBRAIC SOLUTIONS OF POLYNOMIAL SYSTEMS

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This note is based in the results of Chavarriga, Llibre and Sotomayor [1]. By definition a *polynomial system* is a differential system of the form

$$\frac{dx}{dt} = P(x,y), \quad \frac{dy}{dt} = Q(x,y), \tag{1}$$

where P and Q are polynomials with real or complex coefficients; i.e. $P, Q \in \mathbf{R}[x,y]$ or $P, Q \in \mathbf{C}[x,y]$ respectively. We say that $m = \max\{\deg P, \deg Q\}$ is the *degree* of this polynomial system.

Studies of polynomial vector fields were carried out by Poincaré in his memoirs [5]. Poincaré's aim was the global qualitative study on the whole plane of all the solutions of a differential system, and since this is a broad subject he decided to limit it in the first instance to polynomial vector fields.

The algebraic feature of polynomial systems renders natural certain questions and problems of an algebraic or an algebro-geometric nature. Darboux wrote three nice papers on this subject and Poincaré followed him with several of his papers.

The subject of polynomial systems figures also on Hilbert's list of problems [3]. Hilbert formulated his 16-th problem by dividing it in two parts. One about real algebraic curves (or surfaces) and another about the maximum number of limit cycles which could appear in a polynomial system (1).

Problems on polynomial systems (as in many other areas of Mathematics) are usually easy to state and hard to solve and progress in this subject is slow.

In this note we study the link between the integrability of a polynomial system (1) and their algebraic solutions. A key contribution in this direction are Darboux results. Our contribution consists in some results on cases not covered by Darboux Theorem. First we need some preliminary definitions.

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