Journal of Difference Equations and Applications, 2003 Vol. 9 (3/4), pp. 417–422



A Note on the Set of Periods of Transversal Homological Sphere Self-maps

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(Received 2 April 2002; In final form 22 August 2002)

Dedicated to Professor Alexander N. Sharkovsky on the occasion of his 65th Birthday.

Let *M* be a *n*-dimensional manifold with the same homology than the *n*-dimensional sphere. A $C^1 \operatorname{map} f : M \to M$ is called transversal if for all $m \in \mathbb{N}$ the graph of f^m intersects transversally the diagonal of $M \times M$ at each point (x, x) such that *x* is a fixed point of f^m . We study the minimal set of periods of *f* by using the Lefschetz numbers for periodic points. In the particular case that *n* is even, we also study the set of periods for the transversal holomorphic self-maps of *M*.

Keywords: Set of periods; Periodic points; Homological sphere maps; Transversal maps; Holomorphic maps; Lefschetz fixed point theory

(2000) AMS Mathematics Subjects Classification: 55M20; 57N05; 57N10

INTRODUCTION AND STATEMENT OF THE RESULTS

Let *M* be a compact manifold of dimension *n*. Let $f : M \to M$ be a continuous map. A *fixed* point of *f* is a point *x* of *M* such that f(x) = x. Denote the totality of fixed points by Fix(f). The point $x \in M$ is periodic of period *m* if $x \in Fix(f^m)$ but $x \notin Fix(f^k)$ for all k = 1, ..., m - 1. Let Per(f) denote the set of periods for all the periodic points of *f*.

In this paper, we assume that the n-dimensional manifold M has the same homology than the n-dimensional sphere; i.e. the rational homological groups of M are $H_j(M; \mathbb{Q}) = \mathbb{Q}$ if $j \in \{0, n\}$, and $H_j(M; \mathbb{Q}) = 0$ otherwise. Then, a continuous map $f : M \to M$ is called a homological sphere self-map. Then f induces the homology endomorphism $f_{*j}:$ $H_j(M; \mathbb{Q}) \to H_j(M; \mathbb{Q})$ for $j \in \{0, n\}$. It is known that f_{*0} is the identity and that f_{*n} is determined by the image of 1; i.e. $f_{*n}(1) = d$. The integer d is called the *degree* of f. For more details see for instance [20].

Let $f : \mathbb{S}^1 \to \mathbb{S}^1$ be a continuous map. We define the *minimal set of periods of f in the class of continuous self-maps of* \mathbb{S}^1 as the set

$$\operatorname{MPer}_{c}(f) := \bigcap_{g} \operatorname{Per}(g),$$

where g runs over all continuous self-maps of S^1 of the same degree than f.

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ISSN 1023-6198 print/ISSN 1563-5120 online @ 2003 Taylor & Francis Ltd DOI: 10.1080/1023619021000047833