## Open problems on the algebraic limit cycles of planar polynomial vector fields

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**Abstract.** We collect several open problems that have appeared in the study of the algebraic limit cycles of the real planar polynomial vector fields.

Mathematics subject classification: 34C05. Keywords and phrases: Algebraic limit cycle, polynomial vector field, Poincaré problem, invariant algebraic curve.

## 1 Introduction

We divide this brief presentation of several open problems on the algebraic limit cycles of the real planar polynomial vector fields into the following sections:

- 2. Invariant algebraic curves.
- 3. Algebraic limit cycles.
- 4. A unique irreducible invariant algebraic curve.
- 5. Quadratic polynomial vector fields.
- 6. Cubic polynomial vector fields.
- 7. Configurations of algebraic limit cycles.

## 2 Invariant algebraic curves

Since Darboux [12] has found in 1878 connections between algebraic curves and the existence of first integrals of planar polynomial vector fields, invariant algebraic curves are a central object in the theory of integrability of these vector fields. Today after more than one century of investigations the theory of invariant algebraic curves is still full of open questions.

A real planar polynomial differential system is a differential system of the form

$$\frac{dx}{dt} = \dot{x} = P(x, y), \qquad \frac{dy}{dt} = \dot{y} = Q(x, y), \tag{1}$$

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