

ON THE LIMIT CYCLES OF THE LIÉNARD DIFFERENTIAL SYSTEMS

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Abstract

One of the main interesting problems in the qualitative theory of planar differential equations is the problem of studying their limit cycles. For a particular subclass of planar differential systems, the Liénard systems, we shall present some old and new results on their limit cycles.

Key words: *Limit cycles, non-existence, uniqueness, Liénard system*

AMS subject classifications: *34C05, 34C07.*

1 Introduction

One of the most interesting and classical problems in the qualitative theory of planar differential equations is the study of their *limit cycles*, i.e. for a differential system of the form

$$\begin{aligned}\dot{x} &= P(x, y), \\ \dot{y} &= Q(x, y),\end{aligned}\tag{1}$$

where $P, Q : \mathbb{R}^2 \rightarrow \mathbb{R}$ are C^1 functions, what are their isolated periodic orbits in the set of all periodic orbits?

One of the classes of planar differential systems more studied are those equivalent to the *generalized Liénard differential equation*

$$\ddot{x} + f(x)\dot{x} + g(x) = 0,\tag{2}$$

which were considered by many researchers, for instance see the references of this paper, or if the day that I was written this paper you looked in MathSciNet for the number of articles with the keywords *limit cycle* and *Liénard* you obtained 404.

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