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On the integrability of the differential systems in dimension two and of the polynomial differential systems in arbitrary dimension*

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Abstract This is a survey on recent results providing sufficient conditions for the existence of a first integral, first for vector fields defined on real surfaces, and second for polynomial vector fields in $\mathbb{R}^n$ or $\mathbb{C}^n$ with $n \geq 2$. We also provide an open question and some applications based on the existence of such first integrals.

Keywords first integral, 2-dimensional flows, Darboux theory of integrability, invariant algebraic surface, exponential factor.

MSC(2000) 34C05, 34C14, 34C40, 37C10, 34A34.

1. Introduction

In many branches of applied mathematics, physics and, in general, in applied sciences there appear nonlinear ordinary differential equations. If a differential equation or vector field defined in a real or complex manifold has a first integral, then its study can be reduced in one dimension. Therefore a natural question is: *Given a vector field on a manifold, how to recognize if this vector field has a first integral*? In general this question has no a good answer up to now.

In this survey we provide sufficient conditions for the existence of a first integral, first for vector fields defined on two-dimensional real manifolds (or surfaces), and second for polynomial vector fields in $\mathbb{R}^n$ or $\mathbb{C}^n$ with $n \geq 2$. An open question which essentially goes back to Poincaré is presented. Finally some applications on the existence of these kinds of first integrals to physical problems, centers, foci, limit cycles and invariant hyperplanes are mentioned.

The table of contents of the survey is:

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