PERIODIC ORBITS IN THE ZERO–HOPF BIFURCATION OF THE RÖSSLER SYSTEM

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To the memory of Vasile Mioc

ABSTRACT. A zero-Hopf equilibrium is an isolated equilibrium point whose eigenvalues are $\pm\omega i \neq 0$ and 0. For a such equilibrium there is no a general theory for knowing when from this equilibrium bifurcates a small–amplitude periodic orbit moving the parameters of the system. We provide here an algorithm for solving this problem. In particular, first we characterize the values of the parameters for which a zero–Hopf equilibrium point takes place in the Rössler systems, and we find two one–parameter families exhibiting such equilibria. After for one of these families we prove the existence of one periodic orbit bifurcating from the zero–Hopf equilibrium. The algorithm developed for studying the zero–Hopf bifurcation of the Rössler systems can be applied to other differential system in $\mathbb{R}^n$.

1. Introduction and statements of the main result

In 1979 Rössler [25] inspired by the geometry of 3-dimensional flows, introduced several systems as prototypes of the simplest autonomous differential equations having chaos, the simplicity is in the sense of minimal dimension, minimal number of parameters and minimal nonlinearities. In MathSciNet appear at this moment more than 171 articles about the Rössler's systems.

Rössler invented a series of systems, the most famous is probably

$$\begin{aligned} \frac{dx}{dt} &= \dot{x} = -y - z, \\ \frac{dy}{dt} &= \dot{y} = x + ay, \\ \frac{dz}{dt} &= \dot{z} = bx - cz + xz, \end{aligned} \tag{1}$$

introduced in [25], see also [12]. While the Rössler systems were created for studying the existence of strange attractors in differential systems of dimension three, many authors have studied the periodic orbits of these systems depending on their three parameters a, b and c. A brief summary of the results on the periodic orbits of the Rössler systems is done in section 2. The integrability of those systems was studied in [19], see also the references quoted there.

2010 *Mathematics Subject Classification.* Primary 37G15, 37G10, 34C07.
Key words and phrases. Rössler system, periodic orbit, averaging theory, zero Hopf bifurcation.