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A NOTE ON THE DZIOBEK CENTRAL CONFIGURATIONS

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ABSTRACT. For the Newtonian n-body problem in $\mathbb{R}^{n-2}$ with $n \geq 3$ we prove that the following two statements are equivalent.

(a) Let x be a Dziobek central configuration having one mass located at the center of mass.

(b) Let x be a central configuration formed by $n-1$ equal masses located at the vertices of a regular $(n-2)$-simplex together with an arbitrary mass located at its barycenter.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

The main problem of the classical Celestial Mechanics is the *n-body problem*; i.e. the description of the motion of n particles of positive masses under their mutual Newtonian gravitational forces. This problem is completely solved only when $n = 2$, and for $n > 2$ there are only few partial results.

The equations of motion of the Newtonian n-body problem in the d-dimensional space $\mathbb{R}^d$ are

$$\ddot{x}_i = \sum_{j=1,\ j\neq i}^{n} \frac{m_j(x_j - x_i)}{r_{ij}^3}, \quad \text{for} \quad i = 1, \ldots, n,$$

where $x_i \in \mathbb{R}^d$ are the positions of the bodies, $r_{ij} = |x_i - x_j|$ are their mutual distances, and m_i are their masses.

The vector $x = (x_1, \ldots, x_n) \in \mathbb{R}^{nd}$ denotes the *configuration* of the system formed by the n bodies. Clearly the differential equations of motion are well-defined when the configuration is of non-collision, that is when $r_{ij} \neq 0$ for $i \neq j$.

The *dimension* $\delta(x)$ *of a configuration* x is defined as the dimension of the smallest affine subspace of $\mathbb{R}^d$ which contains all of the points x_i. Of course, the configurations with $\delta(x) = 1, 2, 3$ are called *collinear, planar* and *spatial*, respectively. The dimension $\delta(x)$ of a non-collision configuration of $n \geq 2$ bodies satisfies $1 \leq \delta(x) \leq n-1$.

The *total mass* and the *center of mass* of the n bodies are defined as

$$M = m_1 + \ldots + m_n, \qquad c = \frac{1}{M}\left(m_1 x_1 + \cdots + m_n x_n\right),$$

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