Chapter 1

The Averaging Theory for Computing Periodic Orbits

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1.1 Preface

The method of averaging is a classical tool allowing us to study the dynamics of the non-linear *differential systems* under periodic forcing. The method of averaging has a long history starting with the classical works of Lagrange and Laplace, who provided an intuitive justification of the method. The first formalization of this theory was done in 1928 by Fatou [34]. Important practical and theoretical contributions to the averaging theory were made in the 1930's by Bogoliubov–Krylov [8], in 1945 by Bogoliubov [7], and by Bogoliubov–Mitropolsky [9] (english version 1961). For a more modern exposition of the averaging theory, see the book Sanders–Verhulst–Murdock [78].

Every orbit of a differential system is homeomorphic either to a point, or to a circle, or to a straight line. In the first case it is called a *singular point* or an *equilibrium point*, and in the second case it is called a *periodic orbit*. The third case does not have a name. These notes are dedicated to studying analytically the periodic orbits of a given differential system.

We consider differential systems of the form

$$\dot{\mathbf{x}} = F_0(t,\mathbf{x}) + \varepsilon F_1(t,\mathbf{x}) + \varepsilon^2 R(t,\mathbf{x},\varepsilon), \tag{1.1}$$

with $\mathbf{x}$ in some open subset D of $\mathbb{R}^n$, $F_i\colon \mathbb{R}\times D \to \mathbb{R}^n$ of class C^2 for $i=1,2$, $R\colon \mathbb{R}\times D\times(-\varepsilon_0,\varepsilon_0)\to\mathbb{R}^n$ of class C^2 with $\varepsilon_0>0$ small, and with the functions F_i and R being T-periodic in the variable t. Here, the dot denotes derivative with respect to the time t.

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