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Limit cycles in continuous and discontinuous piecewise linear differential systems with two pieces separated by a straight line

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Abstract. This paper is a survey on the study of the maximum number of limit cycles of planar continuous and discontinuous piecewise linear differential systems defined in two half–planes separated by a straight line L. We restrict our attention to the crossing limit cycles, i.e. to the limit cycles having exactly two points on the straight line L. We summarize the results known by now and describe the tools for obtaining them.

Mathematics subject classification: 34C05, 34C07, 37G15.
Keywords and phrases: Discontinuous differential system, continuous differential system, limit cycle, piecewise linear differential system.

1 Introduction and results

Every continuous or discontinuous piecewise linear differential system with two pieces separated by a straight line in the plane $\mathbb{R}^2$ after a linear change of variables can be written into the form

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} a_{11}^- & a_{12}^- \\ a_{21}^- & a_{22}^- \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} b_1^- \\ b_2^- \end{pmatrix} \text{ for } x < 0;$$

and

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} a_{11}^+ & a_{12}^+ \\ a_{21}^+ & a_{22}^+ \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} b_1^+ \\ b_2^+ \end{pmatrix} \text{ for } x > 0.$$

The goal of this paper is to summarize what is known about the following question: *How many limit cycles can such continuous or discontinuous piecewise linear differential systems have*? For continuous piecewise systems both linear differential systems coincides on the straight line $x = 0$, and for the definition of discontinuous piecewise systems we follow the rules of Filippov [9].

The study of the piecewise linear differential systems goes back to Andronov, Vitt and Khaikin [1], and nowadays such systems still continue to receive the attention of many researchers. These differential systems are widely used to model processes appearing in electronics, mechanics, economy, etc., see for instance the books of di