Journal of Computational and Applied Mathematics 277 (2015) 171–191

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Limit cycles for continuous and discontinuous perturbations of uniform isochronous cubic centers

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ARTICLE INFO

Article history:
Received 12 June 2014

MSC:
primary 34A36
34C07
34C25
34C29
34M35
37G15

Keywords:
Polynomial vector field
Limit cycle
Averaging method
Periodic orbit
Uniform isochronous center

ABSTRACT

Let p be a uniform isochronous cubic polynomial center. We study the maximum number of small or medium limit cycles that bifurcate from p or from the periodic solutions surrounding p respectively, when they are perturbed, either inside the class of all continuous cubic polynomial differential systems, or inside the class of all discontinuous differential systems formed by two cubic differential systems separated by the straight line $y = 0$.

In the case of continuous perturbations using the averaging theory of order 6 we show that the maximum number of small limit cycles that can appear in a Hopf bifurcation at p is 3, and this number can be reached. For a subfamily of these systems using the averaging theory of first order we prove that at most 3 medium limit cycles can bifurcate from the periodic solutions surrounding p, and this number can be reached.

In the case of discontinuous perturbations using the averaging theory of order 6 we prove that the maximum number of small limit cycles that can appear in a Hopf bifurcation at p is 5, and this number can be reached. For a subfamily of these systems using the averaging method of first order we show that the maximum number of medium limit cycles that can bifurcate from the periodic solutions surrounding p is 7, and this number can be reached.

We also provide all the first integrals and the phase portraits in the Poincaré disc for the uniform isochronous cubic centers.

1. Introduction and statement of the main results

One of the main open problems in the qualitative theory of planar differential systems is the investigation of the limit cycles that can bifurcate from such systems when we vary the parameters.

A classical way to investigate limit cycles is perturbing a differential system which has a center. In this case the perturbed system displays limit cycles that bifurcate, either from the center (having the so-called Hopf bifurcation), or from some of the periodic orbits around the center, see for instance Pontrjagin [1], the second part of the book [2], and the hundreds of references quoted there. The problem of studying the limit cycles bifurcating from a center, or from its periodic solutions has been exhaustively studied in the last century and is closely related to the Hilbert's 16th problem. Nevertheless, in spite of all efforts, there is no general method to solve this problem.

In the last decades several works about the bifurcation of limit cycles in planar differential systems having a uniform isochronous center have been published see for instance [3–5]. Aside from its importance in physical applications,

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http://dx.doi.org/10.1016/j.cam.2014.09.007
0377-0427/