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Limit cycles and invariant cylinders for a class of continuous and discontinuous vector field in dimention $2n$

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ABSTRACT

The subject of this paper concerns with the bifurcation of limit cycles and invariant cylinders from a global center of a linear differential system in dimension $2n$ perturbed inside a class of continuous and discontinuous piecewise linear differential systems. Our main results show that at most one limit cycle and at most one invariant cylinder can bifurcate using the expansion of the displacement function up to first order with respect to a small parameter. This upper bound is reached. For proving these results we use the averaging theory in a form where the differentiability of the system is not needed.

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1. Introduction

In control theory and in the study of electrical circuits appear in a natural way the continuous and discontinuous piecewise linear differential systems, see for instance [6,9] and the references therein. Such differential systems can exhibit complicate dynamics such as those exhibited by general nonlinear differential systems. The limit cycles and the invariant sets by the flow are some of the main components in the qualitative description of the dynamical behavior of a differential system.

In this paper we study the existence of limit cycles and invariant generalized cylinders for the class of control systems represented by

$$\dot{x} = A_0 x + \varepsilon F(x), \tag{1}$$

where

(i) $A_0 \in \mathcal{M}_{2n}(\mathbb{R})$ with eigenvalues $\{\pm i p_1/q_1, \ldots, \pm i p_n/q_n\}$ where p_k and q_k are positive integers for $k = 1, \ldots, n$ and $(p_k, q_k) = 1$, where $(.,.)$ denotes the greatest common divisor of p_k and q_k.
(ii) $p_k/q_k \neq p_l/q_l$ for $k \neq l$.
(iii) $\varepsilon \neq 0$ is a sufficiently small real parameter.
(iv) $F : \mathbb{R}^{2n} \to \mathbb{R}^{2n}$ given by

$$F(x) = Ax + \varphi_0(k^T x) b, \tag{2}$$

with $A \in \mathcal{M}_{2n}(\mathbb{R})$ and $\varphi_0 : \mathbb{R} \to \mathbb{R}$ is the discontinuous function

$$\varphi_0(y) = \begin{cases} -1 & y \in (-\infty, 0), \\ 1 & y \in (0, \infty). \end{cases} \tag{3}$$

Here the dot denotes derivative with respect to t.

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